***9(i). প্রমাণ কর যে,

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3-1 & y^3-1 & z^3-1 \end{vmatrix} = (xyz-1)(x-y)(y-z)(z-x).$$

(ii). যদি $B = \begin{bmatrix} l & m & n \\ l^2 & m^2 & n^2 \\ l^3-1 & m^3-1 & n^3-1 \end{bmatrix}$

হয়,তবে প্রমাণ কর যে,

$|B| = (lmn-1)(l-m)(m-n)(n-l)$. **(All-18)**

(iii). $\begin{matrix} x+y+z=1 \\ lx+my+nz=k \\ l^2x+m^2y+n^2z=k^2 \end{matrix}$ সমীকরণ জোটটিকে

$AX = B$ আকারে প্রকাশ করে দেখাও যে,

$\det(A) = (l-m)(m-n)(n-l)$ **(J-17)**

(iv). x, y, z এর যে কোনো দুইটি সমান না হলে এবং

$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ হলে, তবে প্রমাণ কর যে,

$(1+xyz) = 0$.

(i). সমাধানঃ $L.H.S = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3-1 & y^3-1 & z^3-1 \end{vmatrix}$

$$= \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} - \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} - \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} + \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= (xyz-1)\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= (xyz-1)\begin{vmatrix} 1-1 & 1-1 & 1 \\ (x-y) & (y-z) & z \\ (x^2-y^2) & (y^2-z^2) & z^2 \end{vmatrix}$$

$[c_1' = c_1 - c_2, , c_2' = c_2 - c_3]$

$$= (xyz-1)\begin{vmatrix} 0 & 0 & 1 \\ (x-y) & (y-z) & z \\ (x+y)(x-y) & (y+z)\ (y-z) & z^2 \end{vmatrix}$$

$$= (xyz-1).1.\begin{vmatrix} (x-y) & (y-z) \\ (x+y)(x-y) & (y+z)\ (y-z) \end{vmatrix}$$

$$= (xyz-1)(x-y)(y-z)\begin{vmatrix} 1 & 1 \\ (x+y) & (y+z) \end{vmatrix}$$

$= (xyz-1)(x-y)(y-z)\{(y+z)-(x+y)\}$

$= (xyz-1)(x-y)(y-z)(y+z-x-y)$

$= (xyz-1)(x-y)(y-z)(z-x)$

$= R.H.S$

(ii). সমাধানঃ Same-(i)

(iii). সমাধানঃ দেওয়া আছে ,

$$\begin{bmatrix} 1 & 1 & 1 \\ l & m & n \\ l^2 & m^2 & n^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$$

$$\therefore \det(A) = \begin{vmatrix} 1 & 1 & 1 \\ l & m & n \\ l^2 & m^2 & n^2 \end{vmatrix}$$

$$=\begin{vmatrix} 1-1 & 1-1 & 1 \\ (l-m) & (m-n) & n \\ (l^2-m^2) & (m^2-n^2) & n^2 \end{vmatrix}$$

$[c_1' = c_1 - c_2 \,\&\, c_2' = c_2 - c_3]$

$$=\begin{vmatrix} 0 & 0 & 1 \\ (l-m) & (m-n) & n \\ (l+m)(l-m) & (m+z)\ (m-n) & n^2 \end{vmatrix}$$

$$=1.\begin{vmatrix} (l-m) & (m-n) \\ (l+m)(x-m) & (m+n)\ (m-n) \end{vmatrix}$$

$$=(l-m)(m-n)\begin{vmatrix} 1 & 1 \\ (l+m) & (m+n) \end{vmatrix}$$

$=(l-m)(m-n)\{(m+n)-(l+m)\}$

$=(l-m)(m-n)(m+n-l-m)$

$=(l-m)(m-n)(n-l)$

$=R.H.S$

(iv). সমাধানঃ দেওয়া আছে ,

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}-xyz\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+xyz\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}=0$$

$$\Rightarrow(1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}=0$$

$$\Rightarrow(1+xyz)\begin{vmatrix} (x-y) & (x^2-y^2) & 1-1 \\ (y-z) & (y^2-z^2) & 1-1 \\ z & z^2 & 1 \end{vmatrix}=0$$

$[r_1' = r_1 - r_2,, r_2' = r_2 - r_3]$

$$\Rightarrow(1+xyz)\begin{vmatrix} (x-y) & (x+y)(x-y) & 0 \\ (y-z) & (y+z)(y-z) & 0 \\ z & z^2 & 1 \end{vmatrix}=0$$

$$\Rightarrow(1+xyz)\begin{vmatrix} (x-y) & (x+y)(x-y) \\ (y-z) & (y+z)(y-z) \end{vmatrix}=0$$

$$\Rightarrow(1+xyz)(x-y)(y-z)\begin{vmatrix} 1 & (x+y) \\ 1 & (y+z) \end{vmatrix}=0$$

$\Rightarrow(1+xyz)(x-y)(y-z)\{(y+z)-(x+y)\}=0$

$\Rightarrow(1+xyz)(x-y)(y-z)(y+z-x-y)=0$

$\Rightarrow(1+xyz)(x-y)(y-z)(z-x)=0$

$\therefore(1+xyz)=0$ **(Showed**

$[\because (x-y)(y-z)(z-x)\neq 0]$