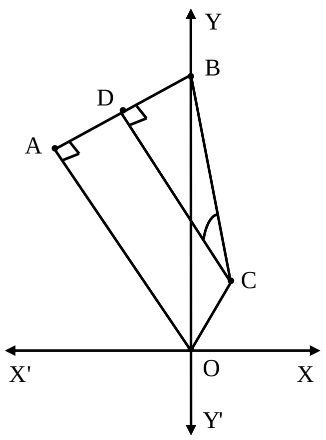
HSC-2021

Subject: Higher Mathematics 1st paper

Assignment-03

Assignment Title: Solution of Straight line related problems by coordinates Geometry

Question:



In the figure, OABC is a quadrilateral. A(– k, 2k), k > 0 and OA = $\sqrt{80}$ unit. The line OC is parallel to the straight line y – 3x = 5 and the point C lies on the perpendicular bisector of the line AB.

a) Find the equation of the straight line AB.

b) Find the coordinates of the point C.

c) In the right angled triangle BCD find the value of tan∠BCD.

d) Find the equation of the bisector of the included acute angle of the straight lines BD and BC.

e) Find the equation of the straight line parallel to the line AB and at a distance $\sqrt{5}$ unit from the point (1, 2).

Solution

a) Coordinates of A is (– k, 2k)

And $OA = \sqrt{80}$

$\Rightarrow \sqrt{(0 + k)^2 + (0 - 2k)^2} = \sqrt{80}$

$\Rightarrow k^2 + 4k^2 = 80$

$\Rightarrow 5k^2 = 80 \Rightarrow k^2 = 16 \quad \therefore k = 4$

$\therefore$ Coordinates of A is (– 4, 8)

Slope of OA is $= \dfrac{8 - 0}{-4 - 0} = \dfrac{8}{-4} = -2$

$\therefore$ Slope of AB is $= \dfrac{1}{2}$ [$\because AB \perp OA$]

$\therefore$ Equation of the line AB is: $y - 8 = \dfrac{1}{2}(x + 4)$

$\Rightarrow 2y - 16 = x + 4$

$\therefore x - 2y + 20 = 0$

b) Equation of the line AB is: $x - 2y + 20 = 0$

At the point B, we have $x = 0$

$\therefore 0 - 2y + 20 = 0 \Rightarrow 2y = 20 \quad \therefore y = 10$

$\therefore$ Coordinates of B is (0, 10)

Again, coordinates of A is (– 4, 8)

$\therefore$ Coordinates of D is $\left(\dfrac{0 - 4}{2}, \dfrac{10 + 8}{2}\right)$ or (– 2, 9)

[$\because$ D is middle point of AB]

Again, equation of the line parallel to the line OC is: $y - 3x - 5 = 0$

$\therefore$ Equation of the line OC is: $y - 3x = 0$ ...... (i)

[Since it passes through the origin]

Again equation of the line DC which is perpendicular to the line AB is:

$2x + y + k = 0$ which passes through D(– 2, 9)

$\therefore 2 \times (-2) + 9 + k = 0 \Rightarrow k = -5$

$\therefore$ Equation of DC is: $2x + y - 5 = 0$ .... (ii)

Solving (i) and (ii) we get, x = 1 and y = 3

$\therefore$ The coordinates of the point C is (1, 3)

c) Coordinates of C is (1, 3)

Coordinates of D is (– 2, 9)

Coordinates of B is (0, 10)

$\therefore$ Slope of CD is, $m_1 = \frac{3-9}{1+2} = \frac{-6}{3} = -2$

Slope of BC is, $m_2 = \frac{3-10}{1-0} = -7$

If the included angle is ϕ then,

$\tan\phi = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \tan\phi = \pm\frac{-2+7}{1+(-2)(-7)}$

$\Rightarrow \tan\phi = \pm\frac{5}{15} \Rightarrow \tan\phi = \pm\frac{1}{3}$

$\therefore \tan\phi = \frac{1}{3}$ [$\because \phi = \angle BCD$ (Acute angle)]

d) Equation of the line BD or AB is:

$x - 2y + 20 = 0$

Again coordinates of B is (0, 10)

Coordinates of C is (1, 3)

$\therefore$ Equation of BC is: $\frac{x-0}{0-1} = \frac{y-10}{10-3}$

$\Rightarrow \frac{x}{-1} = \frac{y-10}{7} \Rightarrow 7x = -y + 10$

$\therefore 7x + y - 10 = 0$

$\therefore$ The equations of the bisector of the angle included between the straight lines BD and BC is:

$$\frac{x-2y+20}{\sqrt{1^2+(-2)^2}} = \pm\frac{7x+y-10}{\sqrt{7^2+1^2}}$$

$$\Rightarrow \frac{x-2y+20}{\sqrt{5}} = \pm\frac{7x+y-10}{\sqrt{5}\times\sqrt{10}}$$

$$\Rightarrow x - 2y + 20 = \pm\frac{7x+y-10}{\sqrt{10}}$$

Here, $a_1a_2 + b_1b_2 = 1\times 7 + (-2)\times 1 = 5 > 0$

$\therefore$ The equation of the bisector of the included acute angle is:

$$x - 2y + 20 = -\frac{7x+y-10}{\sqrt{10}}$$

$$\Rightarrow \sqrt{10}\,x - 2\sqrt{10}\,y + 20\sqrt{10} = -7x - y + 10$$

$$\therefore (\sqrt{10}+7)x + (1-2\sqrt{10})y + (20\sqrt{10}-10) = 0$$

e) Equation of the line AB is: $x - 2y + 20 = 0$

$\therefore$ The equation of the line parallel to AB is:

$x - 2y + k = 0$

The perpendicular distance of the line $x - 2y + k = 0$ from the point (1, 2) is

$$= \frac{|1 - 2\times 2 + k|}{\sqrt{1^2+(-2)^2}} = \frac{|k-3|}{\sqrt{5}} \text{ unit}$$

By the condition, $\frac{|k-3|}{\sqrt{5}} = \sqrt{5}$

$\Rightarrow |k-3| = 5 \Rightarrow k - 3 = \pm 5$

$\Rightarrow k = 3 \pm 5 \quad \therefore k = 8, -2$

$\therefore$ Equation of the required parallel line is:

$x - 2y + 8 = 0$ or $x - 2y - 2 = 0$